Towards a Stable Quantized Convolutional Neural Networks: An Embedded Perspective

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Abstract: Nowadays, convolutional neural network (CNN) plays a major role in the embedded computing environment. Ability to enhance the CNN implementation and performance for embedded devices is an urgent demand. Compressing the network layers parameters and outputs into a suitable precision formats would reduce the required storage and computation cycles in embedded devices. Such enhancement can drastically reduce the consumed power and the required resources, and ultimately reduces cost. In this article, we propose several quantization techniques for quantizing several CNN networks. With a minor degradation of the floating-point performance, the presented quantization methods are able to produce a stable performance fixed-point networks. A precise fixed point calculation for coefficients, input/output signals and accumulators are considered in the quantization process.

1 INTRODUCTION

CNN paradigm architecture has shown a state-of-art performance in the field of image recognition benchmarks (Ciresan et al., 2011). Studying other different aspects of this network would enhance the network use and implementation in embedded devices. The underlying architecture of the CNN might vary from one network to another, and recent architectures have shown large number of layers. Some architectures might include hundreds of layers, millions of coefficients and even require billions of operations (He et al., 2015).

CNN has different types of layers. Convolutional layers are the ones that consume the majority of computation. In these layers multiply-accumulate operations (MACs) are performed extensively, leading to a large number of consumed MACs cycles. While typical applications that use parallel GPUs perform floating-point data format, the embedded systems and specially the real-time ones require a fixed-point format (Dipert et al., 2017).

A key property of CNN is its resistance to noise, while keeping performance stable. Hence, treating any degradation resulted from quantization process as a new source of noise, makes CNN a preferable choice to do this job. In this article, we focus on quantizing the CNN for both coefficients and (input/ output) data with a reduced precision format (8-bits). The quantization process can be applied to both inference and training processes. Training deep neural networks is done by applying many small changes to the network coefficients. These small adjustments utilize higher precision format which accumulate slowly over time and converge to optimal values. There are research efforts to use quantized representations for the training process, but in general more bits are needed for training than for inference (Lin et al., 2016), (Hubara et al., 2016b), (Courbariaux et al., 2014b), (Hubara et al., 2016a), (Gupta et al., 2015), (Esser et al., 2015). Our work focuses on quantization for inference as most real-time embedded systems at the moment will only be deployed with this mode of operations (Dipert et al., 2017).

This article introduces a novel hybrid quantization (HQ) approach. (HQ) quantizes a network layer data points (i.e. both coefficients and data) at different levels. Hence, the network performance stability might be guaranteed using a specific HQ level.

In short, the **Motivation** is: Enhancing the CNN capability to be implemented for embedded devices by reducing the required storage for the network, and the performed computations on these low powered devices. Such enhancement should keep the CNN per-

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formance stable for the inference task. Our **Contribution** is: (1) We generate a stable CNN network with a reduced precision format (8-bits) for both coefficients and (input/ output) data. (2) We apply a hybrid quantization HQ approach. (3) We use a k-means quantization approach which reduces the network computation complexity.

A K-means approach for CNN compression has been used in (Han et al., 2016b). A key difference between this compression method and our work is a dimensionality of the clusters. In (Han et al., 2016b) the clustered points are single coefficients which makes it difficult to achieve efficiency in an embedded processor, especially with vector processors that can handle hundreds of coefficients and data samples in parallel with a single instruction.

The paper is organized as follows. The related works is described in section 2. The floating-point and fixed-point representation and the mapping between them is discussed in Section 3. Additionally it explains the CNN architecture and how complexity of a CNN is measured. Section 4 reviews the hybrid quantization approach. K-means quantization approach is described in Section 6, and Finally conclusion remarks appear in Section 7.

2 RELATED WORKS

Many works have shown CNN quantization (Anwar et al., 2015), (Gysel et al., 2014), (Lin et al., 2016). A common strategy is to quantize all coefficients in a single layer with specified number of bits to represent the integer and fractional parts (Anwar et al., 2015), (Gysel et al., 2014) based on the range of values of the coefficients set. In (Vanhoucke et al., 2011) an 8-bits CNN implementation is compared to an implementation with 32-bits floating-point for speech recognition to speed-up throughput. In (Hwang and Sung, 2014) a fixed-point network with ternary weights and 3-bits activations was presented. Several methods attempt to binarize weights and the activations in neural networks (Hubara et al., 2016a), (Courbariaux et al., 2015), (Soudry et al., 2014), (Rastegari et al., 2016). All of them resulted in significant drop in accuracy relative to floating-point performance. The most efficient among them is the XNOR-Net (Rastegari et al., 2016) architecture which achieves comparable results to floating-point format using only binary weights. Complexity reduction of CNN can be achieved by several other techniques (e.g. SVD reduction, reducing filter sizes) (Han et al., 2016a), (Han et al., 2016b), (Hinton and Salakhutdinov, 2006). SqueezeNet (Iandola et al., 2016) is an example of compressed version of AlexNet (Krizhevsky et al., 2012), reducing 60M parameters to only 0.5M parameters while fully preserving its accuracy. There are also other new approaches for image object recognition like Hierarchical Temporal Memory based on neocortex architecture (Wielgosz and Pietron, 2017), (Wielgosz et al., 2016), (Pietron et al., 2016a), (Pietron et al., 2016b) which is suitable for embedded systems. A K-means approach for CNN compression has been used in (Han et al., 2016b). Recently, many tools have been developed for CNN. Many of them take into account the quantization of the networks. Ristretto tool quantizes CNNs using two phases. In the first phase conventional quantization is used. In the second phase, there is a fine tuning process where the network is retrained using the reduced precision formats of the first phase (Gysel et al., 2014), (Gysel, 2016). The TensorFlow (Google, 2017) learning framework now provides facilities for quantization where floating-point numbers in a certain range are quantized/ dequantized (to/ from) 8-bits precision format.

3 NETWORK QUANTIZATION

Quantization is the process of constraining values from a continuous domain into discrete levels of values. As long as more bits are used in the quantization, as long as the number of levels may increase. In our case, the domain is floating-point representation ($\check{f}$). A floating-point number can be represented as:

$$\check{f} = m \cdot b^e \tag{1}$$

$m \in Z$ is the mantissa, $b = 2$ is the base, and $e \in Z$ is the exponent value. In case of single precision floating point format according to the IEEE-754 standard, the mantissa is assigned 23-bits, the exponent is assigned 8-bits, and 1-bit is assigned for the sign indicator. Therefore, the set of values that can be defined by this format is described by:

$$\check{f}_{\textbf{single}} : \{\pm 2^{-126}, \ldots, \pm(2-2^{-23}) \times 2^{127}\}. \tag{2}$$

Similarly, a reduced precision IEEE-754 mini-float format assigns 10-bits of mantissa, 5-bits of exponent and the sign bit.

Currently, GPUs with parallel processing being applied to machine learning operate mainly using the single precision format. In contrast, embedded DSPs and the latest GPUs operate with 8-bit and 16-bit fixed-point ($\bar{f}$) processing that restricts numbers to the range:

$$\bar{f} : 2^{-\mathcal{F}} \cdot \{-2^{\mathcal{T}-1}, \ldots, 2^{\mathcal{T}-1}-1\} \tag{3}$$

where $\mathcal{T} = \{8, 16\}$ is the total used bits and represents conventional bit-width. $\mathcal{F}$ is a shift down (or up) that determines fractional length, or number of fractional bits. I is the integer length, $I = \mathcal{T} - \mathcal{F} - 1$ for signed numerical representation. When the fractional length varies over individual coefficients or data samples, or over small groups of numbers this format is also referred to as dynamic fixed-point (Courbariaux et al., 2014b), (Gysel, 2016).

Assuming $\mathcal{S}$ is a dataset of points, it is possible to define a general mapping from a floating-point data point $x \in \mathcal{S}$ to fixed-point q as follows (assuming signed representation)

$$q_{\text{fxp}} = Q(x_{\text{flp}}) = \mu + \sigma \cdot \text{round}(\sigma^{-1} \cdot (x - \mu)). \quad (4)$$

To utilize the full range of the input, one can set:

$$\mu = \frac{1}{2}(\max_{x \in \mathcal{S}} x + \min_{x \in \mathcal{S}} x), \sigma = 2^{1-\mathcal{T}} \cdot \max_{x \in \mathcal{S}} |x - \mu| \quad (5)$$

Alternatively, μ can be interpreted and measured as the mean of a distribution of x and σ as the range of the distribution to be represented before saturation. Drawbacks of this approach are the need to calculate and maintain the quantities μ, σ and to apply the normalization procedure in (4).

A second simpler approach uses the magnitude of values and assign int bits I to cover the maximum absolute value such that:

$$I = \lceil (\log_2(\max_{x \in \mathcal{S}} |x|)) \rceil \quad (6)$$

The rest of available bits are assigned to the fraction bits $\mathcal{F}$ and the sign bit.

Yet another approach can define I and $\mathcal{F}$ based on data points distribution histogram. Assuming a given percentage (i.e a large portion of the histogram data points like 99.9 %) to be covered in the quantization assigned I and $\mathcal{F}$. In this case, there will be a small percentage of the data points considered as outliers. To determine the effects of saturation (i.e. outliers), one can experiment the performance with different saturation levels.

3.1 Complexity Reduction Analysis

Typically, the layers of a CNN network have neurons that generate outputs feature maps y_i, $i = 1...N$ as follows

$$y_i = b_i + \sum_{j=1}^{M} (F_{ij} \circ x_j) \quad (7)$$

where F_{ij} is a single two-dimensional (2D) convolutional kernel with dimensions $H \times W$. The operator $\circ$ represents the convolution operation and b_i is the bias vector. Fig. 1 illustrates one three-dimensional (3D) kernel $\bar{F}_i = (F_{i1}, ..., F_{iM})$ with dimensions $H \times W \times M$. Altogether, the N three-dimensional kernels form one four-dimensional (4D) set of filter coefficients.

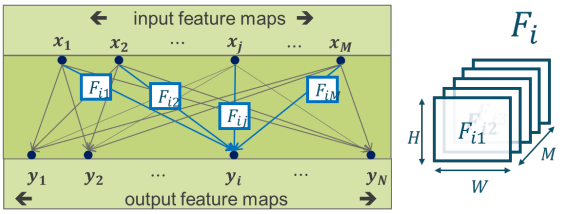


Figure 1: Convolutional layer structure, and the order of a layer kernels in the convolving process.

The number of multiply-accumulate (MAC) operations and cycles spent during the execution of Fig. 1 in a practical implementation is often used as the metric for complexity of a CNN (Courbariaux et al., 2014a). Assuming each output feature map y_i has P output pixels (i.e. image data) the total number of MAC calculations for a convolutional filtering operation is MACs $= (P \cdot H \cdot W \cdot M \cdot N)$.

Complexity may be interpreted in terms of the size of the hardware blocks required to implement the MAC operations or also in terms of the total or average number of cycles required by a processor to perform these operations. In order to compare the reduction in complexity across quantization, we adopt the MAC complexity of 8-bits operations with notation $< \text{coeff}, \text{data} >=< 8b, 8b >$ as a reference unit. Other quantization formats can be compared relative to the complexity of $< 8b, 8b >$ operations. For instance, if 4-bits representation is used for both coefficients and data $< 4b, 4b >$ the relative complexity is approximately $1/4$ that of $< 8b, 8b >$. Similarly, floating-point operations $< \text{flp}, \text{flp} >$ will have approximately $9\times$ to $10\times$ of the complexity of $< 8b, 8b >$ since there are $3\times$ the number of bits to represent the mantissas plus the remaining work required to handle exponents. Table 1 calculates the complexity for the 4^{th} convolutional layer of AlexNet for various quantization formats.

3.2 Convolutional Neural Network Quantization Systems

This section reviews the TensorFlow and Ristretto quantization systems and compares them to the quantization system proposed in this work. The TensorFlow system has an open source quantization module (TensorFlow, 2017). The quantization process uses min, max values (TensorFlow, 2017) to quantize

Table 1: The parameters of 4^{th} AlexNet convolutional layer and its computational complexity in various quantization formats.

Format $< \text{coeff}, \text{data} >$	Relative Complexity [M MACs]
$< 8b, 8b >$	$224 = HPWMN$
$< 4b, 8b >$	$112 = \frac{1}{2} < 8b, 8b >$
$< 4b, 4b >$	$56 = \frac{1}{4} < 8b, 8b >$
$< \text{flp}, \text{flp} >$	$\geq 2016 = 9 < 8b, 8b >$
$N = 256, P = 27^2, M = 48, H = W = 5$	

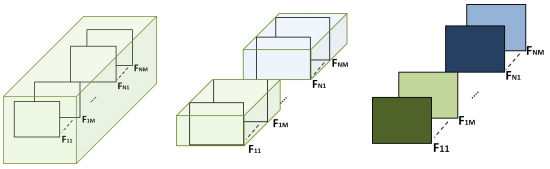
both data and coefficients. The floating-point numbers uses buckets indices as a mapping to quantized values. There is also Dequantization process where the centers of the buckets are mapped back to floating-point value as an approximate estimation for the original values. The range of convolutional layers outputs are estimated analytically.

Ristretto is a system developed by the Laboratory for Embedded and Programmable Systems (LEPs) for University of California, Davis (Gysel et al., 2014). Ristretto is mainly based on dynamic fixed-point and mini-float representation. However, a powers-of-two mode is also available. Ristretto uses fine tuning to recover from any loss in accuracy after its initial trimming phase of quantization. In this quantization schema the quantization is based on ranges (max) of data and coefficients measured empirically. Also, no support for accumulator quantization below 32-bits.

The main advantages of the Ristretto system are its wide range of reduced representation formats and fine tuning approach. The drawback is lack of accumulator quantization. Our solution offers a wide spectrum of layer quantization modes like 4D, 3D and 2D quantization (Hybrid Quantization discussed in the next section) and clustered kernels, or K-means approach that allows advanced quantization based on statistics and automatic tuning. The key feature in TensorFlow is a new reduced format represented by integer values. The drawback is that min and max values must be known at each stage of CNN evaluation.

4 HYBRID QUANTIZATION

This section explores Hybrid Quantization (HQ) of a CNN network. In HQ coefficients are represented in a hybrid fashion by assigning different fixed-point formatting to different partitions of convolutional layers and feature maps. For instance, different precisions (i.e. numbers of bits) and different formats (i.e. I, $\mathcal{F}$) can be assigned to each 2D filter or 3D kernel of a convolutional layer. Fully connected layers are only treated as 4D kernels. Similarly, different precisions and formats can be assigned to different partitions of the input and/or output feature maps. Defining precisions and formats in a hybrid fashion adds overhead for representation both in terms of storage space and facilities to decode it. Nevertheless, it can bring about a significant improvement in CNN accuracy when compared to a homogeneous quantization (i.e. 4D). Varying formats for intermediate accumulated values can be specified in the same hybrid fashion. There are three different formatting schemes are summarized as follows:



(a) 4D-HQ (b) 3D-HQ (c) 2D-HQ

Figure 2: Different quantization schemes.

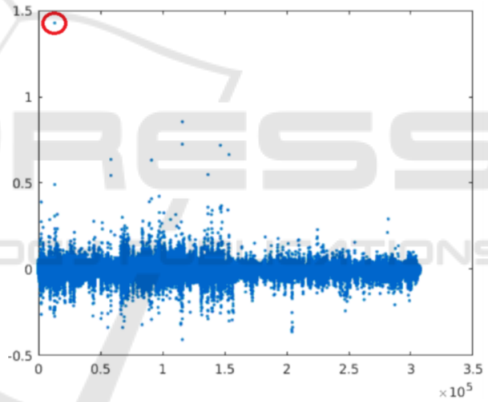


Figure 3: 4D quantization problem when outliers appears in the data (Y axis - magnitudes of coeffs, X axis - id of coeffs).

- **Layer-based or 4D-HQ**: Entire layer kernels are quantized using the same precision format. In Fig. 2(a) all coefficients in the 4D convolutional kernels F_{ij} are quantized by the same $\mathcal{F}, I$ precision format.
- **Kernel-based or 3D-HQ**:

  Coefficients in each 3D kernel in a layer are quantized independently. In Fig. 2(b) each 3D kernel with filters $\bar{F}_i = (F_{i1}, ..., F_{iM})$ is quantized independently with precision format $\mathcal{F}[i], I[i]$ for each i.
- **Filter-based or 2D-HQ**:

  Each 2D filter (each channel in a kernel) in each kernel in a layer is quantized independently. In

Table 2: Comparison of quantization performance for 8-bits AlexNet.

	Format $<$ **coeff**, **data** $>$	**Quantization**	**Top-1 Error**[%]	**Complexity [M MACs]**
Ristretto	$<$ flp, flp $>$		43.1	6516
	$<8b,8b>$	no Fine Tuning	44.0	724
		with Fine Tuning	43.9	724
TensorFlow	$<$ flp, flp $>$		42.2	6512
	$<8b$, flp $>$	4D	46.2	2172
	$<8b,8b>$	4D	51.0	724
Ours	$<$ flp, flp $>$		42.2	6516
	$<8b$, flp $>$	3D	42.3	2172
	$<8b,8b>$	2D	42.7	724
	$<8b,8b>$	3D	42.5	724
	$<8b,8b>$	4D	61.5	724

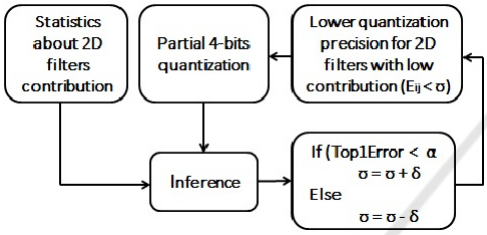


Figure 4: Automatic tuning. α is the reference Top-1 Error. υ is the energy threshold. δ is the energy adjustment amount.

Fig. 2(c) each filter F_{ij} is quantized independently with formatting $\mathcal{F}[i,j], I[i,j]$ for each pair i,j.

For large data precision, homogeneous quantization (i.e. 4D) is sufficient enough to quantize the data points. The problem appears with low precision format, the assigned quantized levels are not sufficient enough to represent all data points in all layer kernels with satisfied accuracy. In 3D quantization, the maximum absolute value in each kernels controls the kernel quantization format. Hence, it reduces the gap between the data points and the maximum absolute value. When using 2D quantization, the quantization produces more efficient quantization, where we restrict the maximum absolute value to each 2D filter independently. This effect is described on Fig. 3. The data point in the red circle (which might seem as an outlier) affects the layer quantization format using the homogeneous quantization (4D). However, using 3D or 2D quantization this effect can be drastically reduced. Therefore kernel quantization (3D) were performed in case of $<8b,8b>$ AlexNet quantization to achieve 1% loss in accuracy in relative to floating-point representation.

Next, we evaluate the performance of various HQ modes and compare against Ristretto and TensorFlow quantization of AlexNet of the the ImageNet validation set (ILSVRC2012 dataset), as shown in Table 2. Our baseline floating-point AlexNet network (i.e. pre-trained network in `MatConvNet` framework) (Vedaldi and Lenc, 2015). This baseline is also ported to a fixed-point C-model. In the C-model, coefficients and input data in each layer are quantized to 8-bits while intermediate accumulator values at the output of a convolutional filter are quantized to 32-bits. Those values are quantized back to 8-bits before being sent to the next layer. Both 3D-HQ and Ristretto experience $<1.0\%$ drop in accuracy relative to the floating-point performance. The quantization in TensorFlow experiences a much higher drop in accuracy (TensorFlow rounding mode), more than 4% despite the use of floating-point to represent data. Our attempts to quantize both data and coefficients in TensorFlow failed to produce reliable results.

Several networks have been tested against our proposed quantization strategy. The goal is to quantize the networks while keeping the performance within the range less than 1% performance degradation relative to the floating-point performance (Table 5). We applied $<8b,8b>$ quantization precision to all networks. Hybrid quantization 4D and 3D are used in the quantization.

5 ADVANCED AUTOMATIC QUANTIZATION

Investigating the performance of mixing 8-bits and 4-bits precision format on different network layers has a lot of insights, specially for embedded devices. Utilizing some statistics gathered during the evaluation of the CNN, we can decide which parts of the network should use (8-bits/ 4-bits) precision format. In this quantization scheme, the energy contribution E_{ij} of each 2D filter F_{ij} is accumulated over a set of probe

Table 3: Hybrid quantization of the 4^{th} layer in AlexNet (40% of coefficients are in 4-bits format), all biases are in 8-bits.

Experiment	Layers: 1, 18	4	7, 9, 11, 14, 16	Top-1 Error [%]	Savings [%]
Conservative Baseline	$<8b,8b>$	$<8b,8b>$	$<8b,8b>$	42.7	0
Moderate	$<8b,8b>$	$<4b,8b>$ (40%)	$<4b,8b>$	43.1	30
Aggressive	$<8b,8b>$	$<4b,8b>$ (100%)	$<4b,8b>$	64.1	44

Table 4: K-means quantization of Convolutional Layer 4 of AlexNet (all other layers are kept in $<$ flp, flp $>$).

Quantization Method	Base Format	Residuals Format	Top-1 Error [%]	Complexity [M Macs]	Layer Savings	Network Savings
2D-HQ Baseline		$<8b,8b>$	42.7	224		
2D-HQ		$<4b,8b>$	65.9	112	50%	15%
K-means	$<8b,8b>$	$<8b,8b>$	42.2	227	-2%	-0.6%
K-means	$<8b,8b>$	$<4b,8b>$	42.7	117	48%	15%

images as follows:

$$E_{ij} = \sum_{\substack{\text{probe}\\ \text{images}}} \sum_{\substack{\text{pixels}\\ p}} |y_{ij}(p)|^2 \tag{8}$$

where $y_{ij} = F_{ij} * x_j$ are partial feature maps. Essentially, the quantization scheme assigns baseline precision (i.e. 8-bits) to 2D filters with high energy contribution and lower precision (i.e. 4-bits) to 2D filters with low energy contribution based on some energy threshold υ. An automatic threshold tuning and quantization procedure was developed to minimize complexity through quantization while maintaining a reference level of Top-1 Error α. The procedure is illustrated in Fig. 4.

Table 3 shows the results of this quantization scheme. The *Conservative* experiment utilizes 8b 2D-HQ quantization throughout all layers. In the *Moderate* experiment, the layers (1, 18) are quantized to $<8b,8b>$. The most computationally expensive Layer 4 is quantized with 40% of its 2D filters in $<4b,8b>$ format and all other layers use $<4b,8b>$ quantization. The *Moderate* experiment suffers $< 1\%$ loss in accuracy relative to floating-point but results in 30% savings in complexity relative to the *Conservative* experiment. The *Aggressive* experiment completely quantizes Layer 4 to $<4b,8b>$ resulting in a drastic loss in accuracy.

6 K-MEANS QUANTIZATION

K-means approach splits a layer coefficients kernels into two parts, basis filters $G_{\ell(i)}$ (i.e. approximations) and residual filters $\tilde{F}_i$ (i.e. differences) leading to $F_i = G_{\ell(i)} + \tilde{F}_i$. The K-means clustering algorithm splits a layer kernels into k centroids kernels serves as approximate kernels for their related cluster groups. The residual kernels are the difference between the original kernels and their corresponding approximation centroids.

The base and residual filters are assigned different quantization formats in order to reduce complexity while maintaining accuracy. In this work we apply K-means clustering to derive a set of $K < N$ basis 3D kernels. These kernels are formatted with $<8b,8b>$ precision. The residual kernels are assigned lower precision, i.e. $<4b,8b>$ or $<4b,4b>$. The final y_i can be described as:

$$y_i = b_i + \sum_{j=1}^{M} \underbrace{G_{\ell(i),j} \circ x_j}_{<8b,8b>} + \underbrace{\tilde{F}_{ij} \circ x_j}_{<4b,8b>,<4b,4b>} \tag{9}$$

where $G_{\ell(i),j}$ are the basis filters assigned to each 3D kernel via some mapping $\ell(\cdot) : [1..N] \mapsto [1..K]$. The residual filters are given by $\tilde{F}_{ij} = F_{ij} - G_{\ell(i),j}$. Assuming the base kernels use $<8b,8b>$ precision and the residuals use $<4b,8b>$, the effective complexity savings of this approach is a factor of $(0.5N-K)/N$. The implementation is as follows: the input is convolved with K basis kernels and generate K temporary feature maps. Next, the output of the residual filters are added to their corresponding base filter temporary feature maps to yield the final output of the convolutional layer. A disadvantage of this approach is that the temporary feature maps have to be stored. Table 4 shows the performance of the K-means approach for convolutional Layer 4 in AlexNet, the layer with the largest complexity (all other layers are kept in floating-point format). While 2D-HQ formatting to $<4b,8b>$ results in 50% reduction in complexity of the layer, the loss in accuracy is drastic. K-means quantization to $<8b,8b>$ format with $K = 4$ is attempted as a baseline (i.e. K is chosen to be 4 based on empirical exper-

Table 5: Comparison of different CNN models using the proposed quantization approach (FLP - floating point, FXP - fixed point).

	Top-5 Error [%]		Top-1 Error [%]		FXP Configuration	
	FLP	FXP	FLP	FXP	Coeff	Data
Caffe-AlexNet	20.33	20.51	43.35	43.64	8b, 4D	8b, 4D
	20.33	20.43	43.35	43.52	8b, 3D	8b, 4D
MatConvNet-AlexNet	19.93	34.28	42.36	58.83	8b, 4D	8b, 4D
	19.93	20.13	42.36	42.53	8b, 3D	8b, 4D
GoogleNet	13.54	13.60	34.89	34.90	8b, 4D	8b, 4D
	13.54	13.58	34.89	35.09	8b, 3D	8b, 4D
Vgg-VeryDeep-19	10.64	10.72	29.48	29.93	8b, 4D	8b, 4D
	10.64	10.61	29.48	29.62	8b, 3D	8b, 4D
Cactus-it6	8.05	9.34	25.26	27.48	8b, 3D	8b, 4D
ResNet-50	8.29	8.80	25.11	26.25	8b, 4D	8b, 4D
	8.29	8.63	25.11	25.78	8b, 3D	8b, 4D
ResNet-101	7.61	8.36	23.96	25.69	8b, 4D	8b, 4D
	8.61	7.91	23.96	24.59	8b, 3D	8b, 4D
ResNet-152	7.23	8.11	23.51	24.96	8b, 4D	8b, 4D
	7.23	8.07	23.51	25.05	8b, 3D	8b, 4D

Table 6: Hybrid clustered quantization of AlexNet.

Base	$< 8b, 8b >$
Residuals	Layer 1: $< 8b, 8b >$ all others : $< 4b, 8b >$
Top-1 Error	44.1%
k - clusters	$k = 2$ for layers 1, 4, 11, 14, 16 $k = 3$ for layer 7 $k = 6$ for layer 9 $k = 4$ for layer 18
Complexity [M MACs]	422
Network Savings	41.6%

iments), yielding a slight increase of 2% in complexity and no significant change in performance. Next, K-means quantization to $<4b, 8b>$ format leads to 48% complexity reduction in the layer and no degradation in accuracy relative to $<8b, 8b>$. In Table 6 we apply K-means quantization to $<4b, 8b>$ to all layers of the network except Layer 1, which is the layer most sensitive to perturbations. The net savings for the network is $> 40\%$ with a degradation in accuracy $< 2\%$. In this experiment we chose different k values for different layers (i.e. based on empirical experiments).

7 CONCLUSIONS AND FURTHER WORK

In this work we presented a novel quantization methods for deep convolutional neural networks. The system reduces memory requirements, area for processing elements and overall power consumption for hardware accelerators. Our research shows that the most popular CNN architectures can be quantized to 8-bits for both coefficients and data outputs with minor degradation in accuracy compared to floating-point.

Hybrid quantization and clustered quantization allow the use of advanced quantization (e.g. $<4b, 8b>$) without significant decrease in performance. Our system is both fast and automated. It was implemented in `MatConvnet` and ported to a C-model. We consider adding new features in the future such as: exploiting other statistics of the network in quantization process, further optimizing the K-means approach, network weights binarization (Rastegari et al., 2016) and (Zhang and Liu, 2016), applying fine-tuning and use re-training in quantization process. These additional features will help to reduce the bit-width even further, and to reduce the computational complexity.

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